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ANALYSIS OF DISPLACEMENTS OF NON-HOMOGENOUS GRANULAR PILE IN HOMOGENOUS GROUND

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ABSTRACT: Granular Piles modified with an anchor at the base called Granular Pile Anchor (GPA) can resist pullout load in addition to compressive loads. Analysis of displacements of GPA whose modulus of deformation increases linearly with depth in homogenous ground is presented. A parametric study quantifies effects of length to diameter ratio, relative stiffness of GP material with respect to that of in situ soil, and degree of non-homogeneity of GPA, on the variations of shear stresses, tip and top displacements of, and of axial force in GPA with depth.

INTRODUCTION

Among the various methods of ground improvement available, granular piles are the most versatile, effective and economical. Granular piles transfer compressive load to deeper depths of the ground through shaft resistance generated along the length of the pile. Granular piles can be made to resist uplift or pullout loads by a simple modification of providing a concrete pedestal, a steel plate or a geogrid at the bottom and attaching a metallic rod or a stretched cable to apply the pullout force at the base of the GPA. The pullout load transmitted to the tip of GPA enables it to resist the uplift force. Phani Kumar (1997) reported tests on models of granular pile anchors to control heave in expansive soils. Lillis (2004) reported results from in situ tests on pullout response of GPA. Kumar *et al.* (1997 & 1999) and Ranjan *et al.* (2000) present results from laboratory and field tests on pullout response of GPA in cohesive and cohesionless soils. Linear and non-linear analyses of displacements of GPA in homogenous and non-homogenous ground are presented by Madhav *et al.* (2008) and Vidyaranya *et al*. (2009).

PROBLEM DEFINITION

A GP of length, L, and diameter, d, with the soil and pile material characterized by moduli of deformation E_s and E_{gp} the latter varying linearly with depth as [$=E_{gp0}(1+\alpha z/L)$], and unit weights of γ_s and γ_{gp}, respectively is considered with pullout load as shown in Figure 1. The Poisson's ratio of the soil is ν_s. A force, P_o, applied at the base of GPA is resisted by shear stress, τ, acting along periphery of pile. The force and stresses acting on and the displacements (upward movements) of GPA are shown in Figure 2(a). The applied force, P_0, is resisted by the interface shear stresses, τ, acting on cylindrical boundary of GPA. The shear stresses vary with depth, z. The displacements increase with depth from a value of ρ_{u0} at the top to ρ_{uL} at the tip. The stresses acting on the in situ soil are shown in Figure. 2(b). To evaluate the upward displacements of the elements of the soil adjacent to the GPA due to the boundary stresses, τ, the GPA surface is divided in

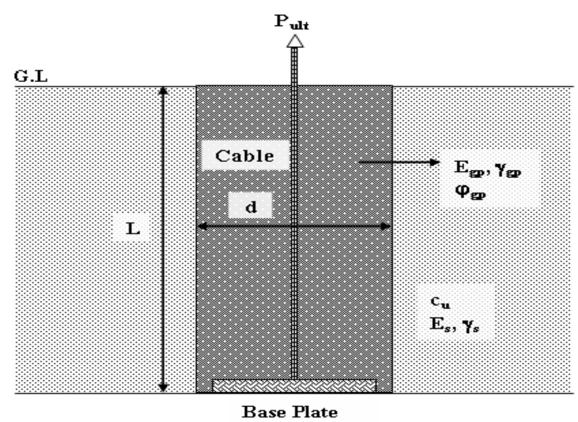
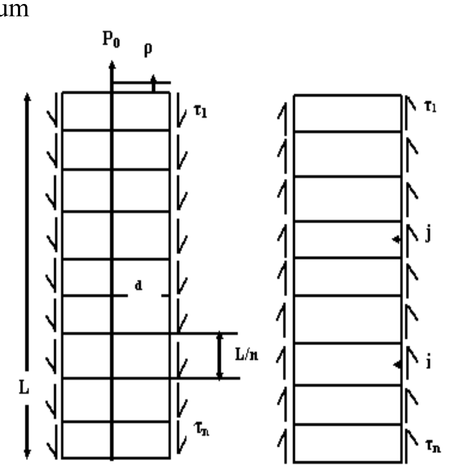


Fig. 1 Schematic of GPA under Pullout.

to 'n' elements of length, L (=L/n). The stress acting on a typical element, j, is τ_j. The displacement, $\rho_{s,ij}$, at the centre of an element, i, due to stresses acting on element, j, are obtained by the procedure described by Poulos and Davis (1980). Integrating numerically, Mindlin's equation(1936) for a point load in the interior of semi-infinite elastic continuum



(a) (b)

Fig. 2 Forces and Stresses acting on GPA and Soil.

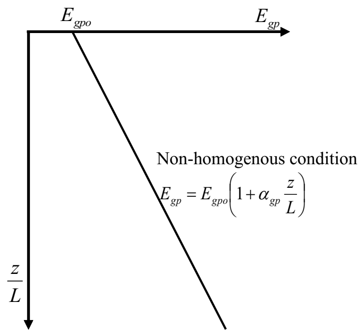


Fig. 3 Variation of deformation modulus of GPA with Depth.

over the cylindrical periphery of the element, the displacement, $\rho_{s,ij}$, is obtained as

$$\rho_{S,ij} = \frac{d}{E_S}.I_{S,ij}.\tau_j \quad (1)$$

where $I_{s,ij}$ – is the soil displacement influence coefficient. The total soil displacement, $\rho_{s,i}$, adjacent to node 'i' due to stresses on all the elements of the GPA, is obtained by summing up all the displacements at node 'i', as

$$\rho_{s,i} = \frac{d}{E_S}\sum_{j=1}^{n} I_{S,ij}.\tau_j \quad (2)$$

The soil displacements at all the nodes are collated to arrive at

$$\{\rho_s\} = \frac{d.[I_s].\{\tau\}}{E_s} \quad (3)$$

where $\{\rho_s\}$ and $\{\tau\}$ are soil displacement and stress vectors of size, n, and $[I_s]$ is the soil displacement influence coefficient matrix of size nxn. The vertical displacements of GPA are obtained considering it to be a compressible pile. The stresses considered on an infinitesimal element of GPA of thickness, Δz. The equilibrium of the forces in the vertical direction reduce to

$$\frac{d\sigma_z}{dz} - \frac{4}{d}\tau = 0 \quad (4)$$

where σ_z is the normal stress in the GPA. The stress-strain relationship for the GPA material, is

$$\sigma_z = E_{gp}.\varepsilon_z = -E_{gp}.\frac{d\rho_{gp}}{dz} \quad (5)$$

where ε_z and ρ_{gp} are respectively the axial strain and displacement of GPA. The modulus of deformation of the granular pile material increases linearly from E_{gp0} at the ground level as

$$E_{gp} = E_{gpo}.\left(1+\alpha_g.\left(z/L\right)\right) \quad (6)$$

where α_g is the non-homogeneity parameter. The pile is homogenous for α_g=0. Combining equations 4, 5 and 6 and simplifying

$$-E_{gpo}\left[\left(\alpha_g/L\right)\frac{d\rho_{gp}}{dz} + \left(1+\alpha_g.\left(z/L\right)\right)\frac{d^2\rho_{gp}}{dz}\right] + \frac{4}{d}\tau = 0 \quad (7)$$

Eq. 7 is solved along with the boundary conditions: at z = 0 (i.e. at the top of GPA) P=0 and at z = L (tip of the GPA), P=P_0 (the applied load). Since Eq. 7 cannot be integrated directly, a numerical (finite difference) approach is adopted. Eq. 7 in finite difference form becomes

$$\{a_i.\rho_{gp,i-1} - b_i.\rho_{gp,i} + c_i.\rho_{gp,i+1}\} - \frac{4L^2}{n^2.E_{gpo}.d}\tau_i = 0 \quad (8)$$

where

$$a_i = \left(1+\alpha_g.z_i' - \frac{\alpha_g}{2.n}\right) \quad b_i = \left(1+\alpha_g.z_i'\right) \quad c_i = \left(1+\alpha_g.z_i' + \frac{\alpha_g}{2.n}\right)$$

where $\rho_{gp,i}$ and τ_i are respectively the displacement at the centre of node 'i' and the shear stress on the interface of element, 'i', of the GPA. Eq. 8 can be written for nodes i = 2 to (n-1). Invoking the first boundary condition, P = 0 implies σ_z = 0 and hence ε_z = 0 leads to

$$\rho_{gp,1} = \rho_{gp,1'} \quad (9)$$

where $\rho_{gp,1'}$ – is the displacement at the imaginary node 1' above the GPA. Eq. 8 can now be written for node 1 as well. The equations for nodes 1to (n-1) are collated and written as

$$[I_{gp}]\{\rho_{gp}\} - \frac{4.L^2}{E_{gpo}.n^2.d}.\{\tau\} = 0 \quad (10)$$

where $[I_{gp}]$ is the GPA displacement influence coefficient matrix, of size nx(n-1).

$$\begin{bmatrix} -c_1 & c_1 & 0 & . & . & . & . & . & . & 0 \\ a_2 & -2b_2 & c_2 & 0 & . & . & . & . & . & 0 \\ 0 & a_3 & -2b_3 & c_3 & 0 & .. & . & . & . & 0 \\ 0 & 0 & a_4 & -2b_4 & c_4 & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & .. \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & a_{n-1} & -2b_{n-1} & c_{n-1} \end{bmatrix} \quad (11)$$

The compatibility of displacement requires

$$\{\rho_S\} = \{\rho_{gp}\} \quad (12)$$

Combining equations 3, 10 &12 one gets

$$([I_{gp}][I_S] - C\{1\})\{\tau\} = 0 \quad (13)$$

where {1} is the unit vector and

$$C = 4.\left(\frac{L}{d}\right)^2 \Bigg/ \left(\frac{E_{gp0}}{E_s}\right).n^2 \quad \text{and} \quad K = \left(\frac{E_{gp0}}{E_s}\right)$$

The 'nth' equation is obtained from the equilibrium of GPA as

$$P_o = \frac{\pi dL}{n}\sum_{i=1}^{n}\tau_i \qquad (14)$$

$$\{1\}\{\tau\} = \tau_{avg}.n \qquad (15)$$

where $\tau_{avg} = P_0/(\pi dL)$ – the average shear stress and {1}-a unit row vector. Equations 13 and 15 are combined to get 'n' equations for the 'n' unknown shear stresses, {τ}. They are solved by either matrix inversion or by Gauss-Seidel methods to get the shear stresses. Knowing the shear stresses, the displacements, $\{\rho_s\}$ and hence $\{\rho_{gp}\}$ are obtained from Eq. 2.

RESULTS AND DISCUSSION

Numerical experimentation or parametric study is carried out for the following ranges of parameters: L/d: 5 to 50, K: 10 to 500; ν_s: 0.5 & α_g= 0 to 1.0.

Shear stress variation with depth with the effect of the non-homogeneity parameter, α_g, of the granular pile material, is shown in Figure 4 for L/d=10, υ_s=0.5 & K=50. The normalized shear stress decreases with depth to a value of about 0.3 near the top (0.1z/L) of GPA and increases to a maximum of 1.8 at the bottom. The pattern of shear stress variation with depth is similar to that noted for incompressible pile (Poulos and Davis 1980). The shear stresses increase marginally with α_g.

The variations of shear stress with depth for GPA for different L/d for K=50, υ_s=0.5 & α_g = 0 (homogeneous GPA) and 0.5 is shown in Figure 5. The non-homogeneity parameter, α_g has a marginal or no effect on the shorter GPA (L/d <10) but for longer GPA the difference is moderate at mid depth. The shear stresses decrease with increase in the non-homogeneity of the pile material. The normalized shear stresses are of the order of 0.4 to 0.6 in the top half and increase sharply near the toe of the GPA for short GPA (L/d=5). For long GPA, i.e., for L/d = 25 and 50, the shear stresses are nearly constant or increase marginally over the top half of the GPA (z/L<0.5) but increase sharply at points near the tip of the GPA. For L/d = 50, while the minimum value of the normalized shear stress, $\tau^* < 0.1$ near the top the maximum value is as large as 3.0 near the tip of the GPA. Figure 6 compares shear stress variations with depth for L/d=10, υ_s=0.5 & α_g=0.5 for different K. Shear stress increases with depth but decreases with increase in α_g. The shear stresses variations in GPA at the top are very small in case of highly compressible granular material. The curve corresponding to K=10 shows very small values of shear stresses in the depth range of z/L<0.65. The shear stresses

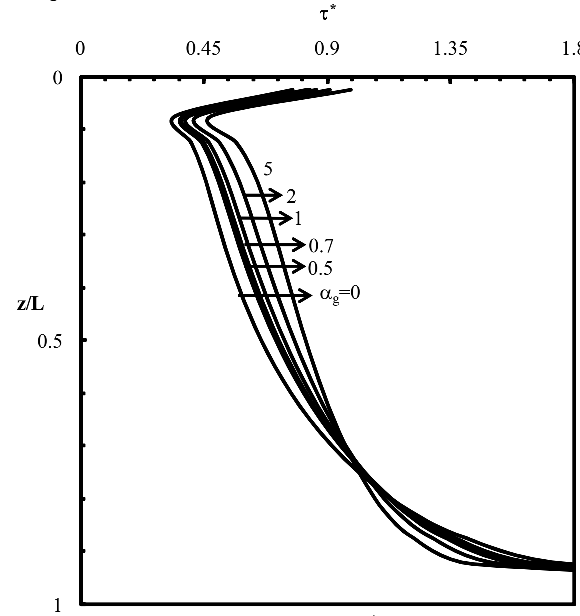


Fig. 4 Variation of shear stress, τ^* vs. z/L for υ_s= 0.5, K=50 & L/d=10 - Effect of α_g.

increases to very large value as z/L tends to 1.0. The shear stresses increase in the top half of the GPA with increasing values of relative stiffness factor, K, while decreasing with K in the lower one-third of the GPA. The shear stresses for K > 500 are almost identical to those corresponding to incompressible pile.

The variations of the displacement coefficient, I_u, with depth for different L/d for K=50, υ_s=0.5 & α_g=0.5 are shown in Figure 7. The displacements increase with increasing L/d and α_g. The displacements are more uniform and increase gradually with depth for short GPA, the top displacements decreasing and the tip displacements increasing sharply with

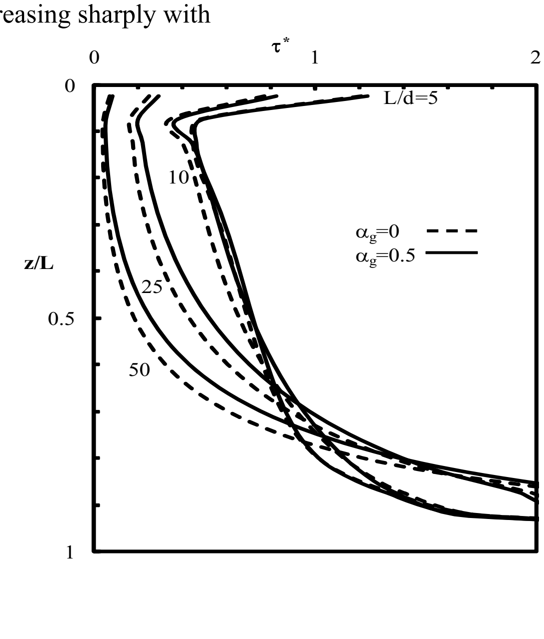


Fig. 5 Shear stress, τ^* vs. z/L for υ_s= 0.5 & K=50 - Effect of L/d & α_g.

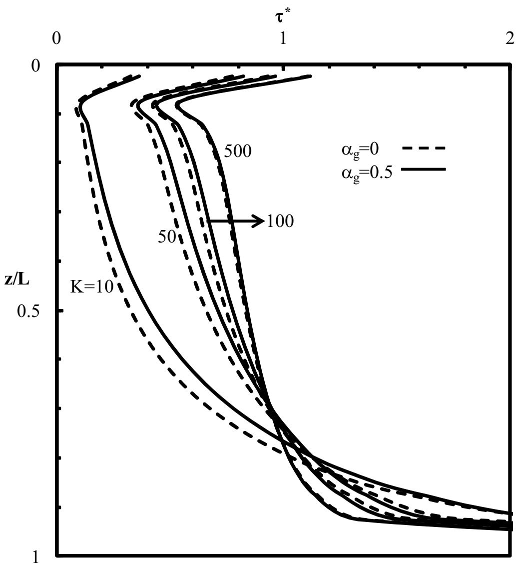


Fig. 6 Variation of shear stress, τ^* vs. z/L for υ_s= 0.5 & L=10 - Effect of K & α_g.

increasing L/d ratios. For GPA, more significant are the decreases in displacements towards the top. Fig. 8 compares the variations of P^* with depth for increasing L/d for homogenous and non-homogenous GPA. The variation of P^* is large for varying α_g at larger L/d but become insignificant at smaller L/d signifying the behavior similar to rigid pile. The normalized uplift force values, P^*, are 40%, 30%, 20% & 5% at mid depth for L/d ratios of 5, 10, 25 & 50 respectively. It implies the applied force is mostly transferred to the in situ soil at great depths in case of longer GPA. Thus GPAs are effective as they transfer the applied load to greater depths.

CONCLUSIONS

An analysis of the GPA is presented considering the deformation modulus of the GPA to increase linearly with

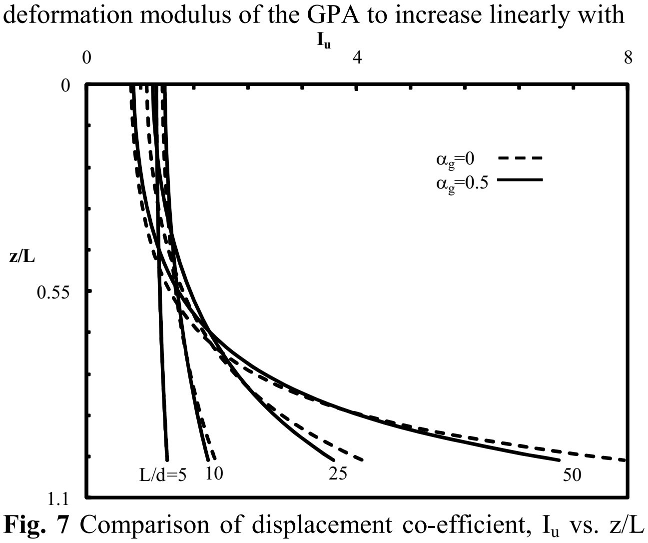


Fig. 7 Comparison of displacement co-efficient, I_u vs. z/L for υ_s= 0.5 & K=50 - Effect of L/d & α_g.

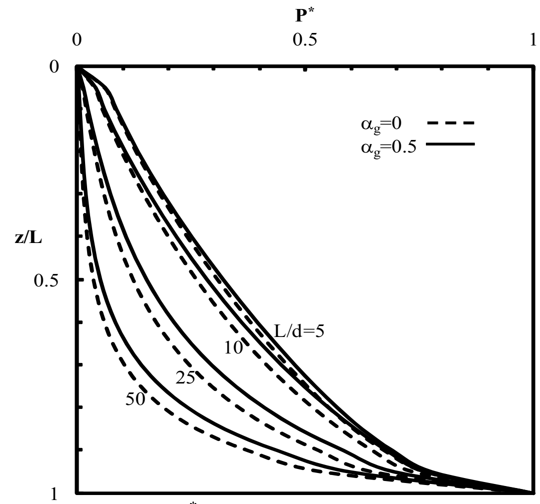


Fig. 8 Pullout load, P^* vs. z/L for υ_s= 0.5 & K=50 - Effect of L/d & α_g.

depth as $[=E_{gp0}(1+\alpha_g z/L)]$. A parametric study has been carried and results are presented in the form of variations of normalized shear stress, displacements and normalized axial pullout force with depth, non-homogeneity parameter, α_g and relative stiffness factor, K. The salient conclusions are

1. GPA's are very effective in transferring the applied loads to strata at depth particularly if they are long and/or if they are relatively more compressible.
2. GPA with modulus increasing with depth behaves more like an incompressible or rigid pile at higher L/d.

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